Weekly Homework 2

Math 485

October 8, 2013

1 Textbook (Stampfli and Goodman)

A. Section 3.3, Page 54: 1,2,3,4.
B. Section 3.4 Page 58: 1,2,3.
C. Section 3.5 Page 61: 1,2,3.
D. Repeat Page 61: 1,2,3 for Asian option.

2 Additional problems

1. (Extra credit - 5 pts) Show that in a Binomial model, the value process V_k^{put} for $0 \leq k \leq n$ of an American put option is a super-martingale under the risk neutral probability.

2. Consider a Binomial model with $S_0 = 81, u = \frac{4}{3}, d = \frac{2}{3}, r = 0, n = 4$. Compute the replicating portfolio for a European call option with strike price $k = 30$, expiration time $n = 4$. (You can either draw a tree and list the portfolio composition at each node, or you can give the answer in the form $\Delta_2(ud) = x, b_2(ud) = y$, for all possiblities of events at each time k).

3. Let X_i be i.i.d. with distribution

$$\begin{aligned} X_i &= 1 \text{ with probability } \frac{1}{2} \\ &= -1 \text{ with probability } \frac{1}{2} \end{aligned}$$

. Classify whether the following is a martingale, sub-martingale, super-martingale with respect to the filtration $\{\mathcal{F}_i^X\}$, or neither. Justify your answer.

a) S_k^1, where $S_k^1 = \sum_{i=0}^k X_i$.

b) $(S_k^1)^2$.
c) S_k^2, where $S_k^2 = \sum_{i=0}^{k} \sin(X_i)$.
d) S_k^3, where $S_k^3 = \sum_{i=0}^{k} \cos(X_i)$.
e) S_k^4, where $S_k^4 = \sum_{i=0}^{k} (-1)^i X_i$.